Solve for $|x+1| = 2|x-1|$?

Solution:

This is a direct example to solve an **Absolute value equation of the form $|a| = |b|$**

Before we get into solving it, why don't we have a look at **some GMAT standard models tested through Absolute Value Equations?**

One of the general models tested on GMAT is $|a| = k$ where k is a constant (Ex- $|a-3| = 6$))

A second model of testing would be that of using a double mod. $||a|-p| = q$ where p and q are constants.

And the third model would be $|a| = |b|$

Each of these cases has a separate way of solving but the **fundamental properties of Absolute Values are always used to solve them.**

Now, that we know we are going to use the third model here, lets deep dive into solving it.

$|x+1| = 2|x-1|$

Since both LHS & RHS contain modulus, lets try to break it and reach to the first simple model of $|a| = k$

Let us substitute $2|x+1|$ as k and hence we have with us

$x+1 = k$ when $x >= -1$

and

$x+1 = (-k)$ when $x < -1$

This is using the **Fundamental properties of Modulus and their basic operations.**

Substitute k as $2|x-1|$ in both the equations

So how have we broken this by now?

$x + 1 = 2|x-1|$ for $x >= -1$

and

$x + 1 = -2|x - 1|$ for $x < -1$

1st case-

$x + 1 = 2|x-1|$ for $x >= -1$

Using the Mod operations, let's have the reference ranges for x and they are

$x + 1 = 2\,|x-1|$; for $x >= -1$

and now lets break the second mod function

We have now, $x + 1 = 2\,(x-1)$ for $x >= -1$ and $x > =1$ which gives us a **combined range of $x > = 1$**

Solving this, $x + 1 = 2x - 2$ or $x = 3$

Is this value in the reference range of $x > = -1$?

YES.

3 is $> =1$ and hence it's a valid solution point. Accepted value of $x=3$.

Now let's see if there are any other values too!

$x + 1 = -2\,(x-1)$ for $x >= -1$ and $x < 1$ which gives us a combined range of $-1 <= x < 1$

Solving this $x + 1 = -2x + 2$ or $3x =1 => x =1/3$

Is $1/3$ between -1 and 1 ?

YES.

Hence it's a valid solution point. Accepted value of $x= 1/3$ also.

2nd Case-

$x + 1 = -2\,|x - 1|$ for $x < -1$

Using the Mod operations, let's have the reference ranges for x and they are,

$x + 1 = -2(x-1)$ for **$x < -1$ and $x >=1$.**

Is it possible for x to be greater than or equal to 1 and also less than -1? **NO.**

Discard this scenario.

Also, we have one more step to go!

$x + 1 = - (-2(x-1))$ for $x < -1$ and $x <1$ which gives us a **combined range of $x < -1$**

Solving we have, $x + 1 = 2x - 2$

Or

$x =3$ but is this value appropriate for the range of $x <-1$? Is $3<-1$? **NO**

So discard $x=3$ for this range.

However, according to the solution above, **you have $x = 3$ and $x = 1/3$ as two solution points for x.**

So we hope by now we hope you have gained the insights on how to solve Absolute value equations of this form.

This approach is explained so that you can build up on the fundamentals of Absolute Value and the methods of opening the mod equations and arriving at values using the reference ranges.

Alternately you can use the method of squaring!

$|x+1|=2|x-1|$

The alternate definition of $|x|$ is the square root of x^2

or

$|x| = \sqrt{x^2}$

So, $|x+1| = \sqrt{(x+1)^2}$ and $|x-1| = \sqrt{(x-1)^2}$ and hence we have the converted equation as

$\sqrt{(x+1)^2} = 2\sqrt{(x-1)^2}$

So, **Squaring both sides,**

$\Rightarrow (x+1)^2 = 4(x-1)^2$

$x^2+2x+1=4x^2-8x+4$

$\Rightarrow 3x^2-10x+3=0$

$\Rightarrow 3x^2 - 9x - x - 3 = 0$

or

$\Rightarrow 3x(x-3) - 1(x-3) = 0$

$\Rightarrow (3x-1)(x-3) = 0$

$\Rightarrow x = 1/3, x = 3$

You can also have a look at this video on Absolute Values to clear and build on your concepts of Absolute Values.

https://www.youtube.com/watch?v=yGBAIcBhQdM

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