

Number Properties Red Book

Factors and multiples are essentially opposites.

If two numbers have a common divisor, their SUM or DIFFERENCE is also divisible by that divisor.

Primes are greater than 1. Smallest prime is 2. First ten primes are: 2,3,5,7,11,13,17,19,23,29.

Prime box: If you can make a number from a another numbers prime box, that number is a factor of the number you used to make the prime box.

GCF: The largest number by which two integers can be divided.

LCM: The smallest number that is a multiple of two integers.

To find the GCF: find the product of the common prime factors, using the lower power of the repeated factor. To find the LCM find the product of all the prime factors of both numbers, using the higher power of the repeated factor.

$1^3 = 1$	$\sqrt[3]{1} = 1$
$2^3 = 8$	$\sqrt[3]{8} = 2$
$3^3 = 27$	$\sqrt[3]{27} = 3$
$4^3 = 64$	$\sqrt[3]{64} = 4$
$5^3 = 125$	$\sqrt[3]{125} = 5$

$1^2 = 1$	$\sqrt{1} = 1$
$1.4^2 \approx 2$	$\sqrt{2} \approx 1.4$
$1.7^2 \approx 3$	$\sqrt{3} \approx 1.7$

General Memorization

Divisible by:

2 if integer is EVEN
3 if sum of digits is div by 3
4 if divisible by 2 twice
5 if ends in 0 or 5
6 if divisible by 2 AND 3
8 if divisible by 2 THREE times
9 if sum is divisible by 9
10 if ends in 0

Add 2 odds or add 2 evens and EVEN you will see. But and an odd with an even, and how ODD it'll be. Just one EVEN number in a multiplication set, an EVEN you will get.

All primes are odd, except 2. If you see a sum of two primes that is odd, one of those primes must be 2.

Consecutive Integers

When dealing with consecutive integers, add one if it's inclusive.

To find the sum of consecutive integers: Average the first and last term, multiply by the number of terms (add 1 if inclusive).

Special sums rule: For any set of consecutive integers with an odd number of terms, the sum of all the integers is always a multiple of the # of terms. That is, the sum of 3 consecutive numbers will be divisible by 3. This ONLY works with an odd number of terms in a consecutive set. That is, the sum of 4 consecutive numbers is NOT divisible by 4. However, the product of any number of consecutive terms will always be divisible by n whether the number of terms is odd or even. That is, the product of 3 or 4 consecutive terms will be divisible by 3 or by 4 respectively. But the SUM of those terms will not be divisible by 4

Powers and Exponents

Powers operate as DECREASING mechanisms on positive fractions.

When raising a power to a power, combine exponents by multiplying. Working with roots: you can split the $\sqrt[3]{(25 \cdot 16)}$ into $\sqrt[3]{25} + \sqrt[3]{16}$ but you cannot split $\sqrt[3]{25 + 16}$.

SAME BASE	SAME EXPONENT
When multiplying expressions with the same base, ADD the exponents first. $(3^2)(3^3) = (3)(3)(3)(3)(3) = 3^5$	When multiplying expressions with the same exponent, MULTIPLY the bases first. $(3^3)(5^3) = (3)(3)(3)(5)(5)(5) = (15)(15)(15) = 15^3$
When dividing expressions with the same base, SUBTRACT the exponents first. $\frac{3^5}{3^2} = \frac{(3)(3)(3)(3)(3)}{(3)(3)} = (3)(3)(3) = 3^3$	When dividing expressions with the same exponent, DIVIDE the bases first. $\frac{9^3}{3^3} = \frac{(9)(9)(9)}{(3)(3)(3)} = (3)(3)(3) = 3^3$

Equations, Inequalities and VICs

You can solve equations by substitution or by combination.

Remember: The GMAT likes to trick you into thinking you always need 3 equations with 3 unknowns. You may not! Watch out.

Manipulating Eq:

M: Multiply

U: Unsquare

D: Distribute the Factors

Equations with absolute values have two solutions. Solve for both the positive and negative solutions. The same is true of even exponent solutions. They are dangerous! Watch out for $x^2 + k = 0$ - these have no real solutions.

Odd exponents only have one solution. Even exponents have 2.

Always make sure that problems that have exponents on both sides share the same base or the same exponent!

To eliminate a root, square both sides.

When solving VICs, try numbers, but you must test every answer choice, even if you found one that works!

General Memorization

Know the favourite quadratics:

$$x^2 - y^2 = (x+y)(x-y)$$

$$x^2 + 2xy + y^2 = (x+y)(x+y)$$

$$x^2 - 2xy + y^2 = (x-y)(x-y)$$

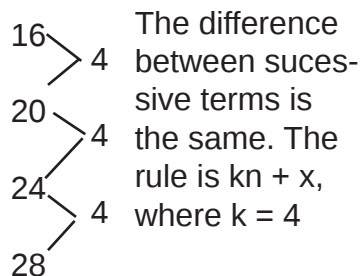
First instinct should be to factor!

Formulas & Sequences

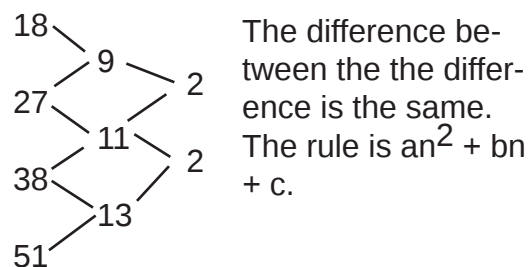
With strange symbol formulas, always do the part in a parentheses first, then work out. Formulas with unspecified amounts should be solved by PICKING NUMBERS.

Solving sequences: It's easier to look for patterns. So, if a question asks $S_n = 3^n$ what is in the units digit of S_{65} ? There must be a pattern, look for it!

If you cannot find it, you can always work through it the hard way:



$$4(1) + x = 16$$
$$x = 12$$



$A = 1$, $B = 6$, $C = 11$ - solved by using the formula for $n = 1, 2, 3$ and then solve the three equations

Inequalities

When you multiply or divide an inequality by a negative number, the inequality sign FLIPS. Do not multiply or divide inequalities by unknowns (unless you know the sign of the unknown)!

Solving with absolute values: to solve $|x - 5| < 8$, solve $x - 5 < 8$ first, then reverse the signs in the brackets and solve: $-x + 5 < 8$

When combining inequalities, line up the variables in a line. When testing values, always try the extremes of a given problem.

Word Translations (Part I)

Working with Ratios:

Label each part of the ratio, set it up as a fraction.

If need be, find the unknown multiplier. For example: A recipe calls for amounts of lemon juice, wine and water in the ratio of 2:5:7. If all three combined yield 42 mill how much wine was included?

$$2x + 5x + 7x = 42$$

$$x = 3$$

$$\text{Wine} = 3(5) = 15$$

All of the following are examples of ratios: "3 to 4" or "3:4" or "3 / 4" as a fraction.

Simple ratio problems can be solved by a fraction proportion. For example: If the ratio of boys to girls is 4 to 7 and there are 35 boys in the class how many girls are there?

$$\begin{array}{rcl} 4 \text{ girls} & & x \text{ girls} \\ \hline 7 \text{ boys} & = & 35 \text{ boys} \end{array}$$

Solve for x.

Population Problems

The key to population problems is to remember to include NOW in your chart as in the example to the right.

In this example, the population doubles every ten minutes, but don't forget to account for right now in solving!

Time Elapsed	Population
20 minutes ago	100
10 minutes ago	200
NOW	400
in 10 minutes	800
in 20 minutes	1600
in 30 minutes	3200

Solving Age Questions

Set up a table to solve age questions:

	5 years ago	NOW	10 years from now
George	G - 5	G	G + 10
Sarah	S - 5	S	S + 10

Rates And Work

$RT=D$ or $RT=W$ where D is distance, W is work. Always match units! That is, if R is expressed in minutes, and T in seconds, either convert R to seconds or T to minutes!

You cannot just average rates! You must find total distance, total time and find the associated rate. Do this by setting up a matrix:

	GOING	RETURN	TOTAL
R	4	6	NOT 5!
T	3	2	5
D	12	12	24

Notice how the correct answer is not the average of 4 and 6!

Careful with time relations. Consider who is traveling longer - this person gets a variable PLUS a value. I.e. If Train A leaves 10 minnutes after Train B, Train A's time is t minutes, and B is t + 10 because train b is travelling for a longer time!

Problems where people are working together, make sure you add their rates!

Word Translations (Part II)

Combinatorics

Combinatorics are solved through factorials. Always divide the factorial by the number of repeating letters in your anagram.

A	B	C	D	E	F	G
1	2	3	4	N	N	N

Would equal $7!$ divided by $3!$ (due to the repeating Ns)

The key to solving these types of questions is to ask yourself if the order matters: Use numbers to represent people or objects when the order matters, use Y and N to represent people and objects when the order does NOT matter.

A	B	C	D	E	F	G
Y	Y	Y	Y	N	N	N

In this example, you would divide $7!$ by $(4! * 3!)$ as the Y's repeat 4 times ($4!$) and the N's repeat 3 times ($3!$).

Probability

1 is the greatest probability, 0 is the least probability. Probability is generally defined as the number of winning outcomes divided by the number of possible outcomes.

Differentiating AND vs OR: To determine the probability of two independent events, to determine the probability of X and Y occurring, multiply their probabilities. To determine the probability of X or Y occurring, add their probability - but only if they are mutually exclusive, that is, X and Y cannot occur. (I.e. What is the probability a die lands on 4 or 5) - then add their individual probabilities.

If X and Y are NOT mutually exclusive (both events can occur), Add the probabilities that each event occurs, then subtract the probability that both events occur together (that is $\text{prob A} + \text{prob B} - (\text{prob A} * \text{prob B})$).

Don't forget about the 1 - x probability trick! Probability that an event will happen + probability it will not = 1.

Don't forget the Domino effect - that is, the outcome of the first event will change the probability of a second event. (I.e. 10 candies in a jar, what is the probability of picking a red candy in your first 2 tries?)

Word Translations (Part III)

Averages, Standard Deviation and Statistics

Average is defined as: $A = S / N$, where a = average, s = sum, n = number of terms. For more complex problems, you may have to set up two average formulas - a before and an after to solve for one of the variables.

Also, note that the average of any evenly spaced set (i.e. $\{ 2, 4, 6 \}$ or $\{9,18,27,36\}$) is always going to be the sum of the first and last number divided by 2. This is true for both odd numbered and even numbered sets, provided they are evenly spaced. The median of is the middle number of a set ordered from least to greatest. In the set $\{ 17, 17, 24, 25, 28 \}$ the median is 24. Sets with an odd number of terms will have a median listed in the set (i.e. 24), sets with an even number of terms take the average of the two most middle numbers.

Standard deviation is a measure of the “spread” of a series of numbers. The closer the numbers, the smaller the SD. The more spread out, the bigger the SD. If you are asked to calculate two or three standard deviations from a number, and are given the SD, just multiply the SD by that frequency then add and subtract from whatever number. I.e. If the mean of a data set is 75, and the standard deviation is 10, the range of values in one standard deviation is 65 - 85.

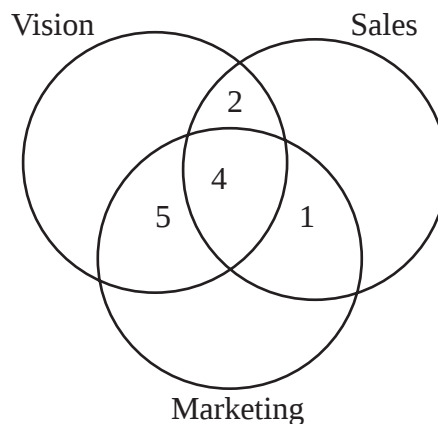
Double and Triple Overlapping sets

Solving for overlapping sets is easy with a double set matrix. Set up as follows:

	A	NOT A	TOTAL
B			
NOT B			
TOTAL			

When working with percents, use 100 for the TOTAL/TOTAL cell, and solve using simple numbers. Careful when the questions require algebraic considerations, for example: “ 10% of all aliens are capable of intelligent thought and have more than 3 arms, 75% of aliens with 3 arms or less are capable of intelligent thought. If 40% of all aliens are capable of intelligent thought, what percent of aliens have more than 3 arms?” (see pg 102 in Red book)

3 Set problems should be solved with venn Diagrams and solved from the inside out. Don't forget to subtract items that intersect in all three spots from items that intersect in two places. I.E: 20 people are on marketing team, 30 on sales, 40 on vision. 5 are on both marketing and sales, 6 on sales and vision, 9 on both marketing and vision, 4 in all three. How many workers are there in total?



Reading Comprehension

Diagramming

Diagram all your questions into Premises (P) and a Conclusion (IC or EC or both). EC are in the question. Order the P and C logically by numbering them (do not rewrite them). Use arrows to identify cause and effect. Underline key words - especially words that limit such as "some", "a few", "lowers", etc.

Skeletal Sketch (Long and Short)

Long passages > 50 lines of text. Short passages average between 30 and 45 lines. Rule of thumb: Number of questions * 2 = time you should spend sketching AND answering the questions. You will see the number of questions listed as "1 of 6"

[insert the box that breaks down the verbal here]

Skeletal Sketch for long readings:

- (1) Detailed information on the first paragraph.
- (2) The topic sentence of each subsequent paragraph
- (3) Key words from each paragraph other than the first, including names, places and terms. DO NOT READ THE WHOLE PARAGRAPH.

Detail Sketch for short readings:

- (1) Read the topic sentence of the first paragraph and summarize it, and underline it.
- (2) Summarize each sentence under the topic sentence of each paragraph. These should be supporting bits of information that add up to the topic sentence.
- (3) Repeat for other paragraphs

General Strategies to remember:

- (1) Justify every word in your answer choice.
- (2) Avoid EXTREME words
- (3) Infer as little as possible
- (4) Preview the first question!
- (5) For general questions, use a point system (2 points for first para, 1 for subsequent)

For specific questions:

- (1) Match key words in the passage
- (2) Defend your answer choice with 1 or 2 sentences around the key word

Remember: The GMAT doesn't usually ask you to actually infer anything, merely restate!

Sentence Correction (Part I)

The 3 C's

- (1) Correctness - is it grammatically sound?
- (2) Concision - is it concise?
- (3) Clarity - is the sentence clear?
 - (1) WORD PLACEMENT - moving words around can change the meaning!
 - (2) KNOWN vs. UNKNOWN - will vs may, must vs should, could, ought, can, would, etc.
 - (3) MULTIPLE MEANINGS - i.e. the shirt is light (color or weight?)
 - (4) SUCH AS VS LIKE - Such as is an example, like is a similarity. They are not the same!

Subject Verb Agreement

Don't get confused by subjects followed by the word of. Of is just a middle man. If you can remove a phrase from the sentence and the sentence makes sense, you probably found a middle man.

The word "And" itself can change a verb from singular to plural, but additive phrases like "along with, in addition to, as well as, accompanied by, together with, including" do not form compound subjects and the verb tense does not change.

Dealing with "or", "either or" and "neither nor" - find the subject that is NEAREST to the verb and make sure that the verb agrees to that. I.e. Neither Joe nor his friends ARE going to the beach. Neither his friends nor Joe IS going to the beach.

Collective nouns are singular. That is, words like "crowd, faculty, team" are singular. Indefinite pronouns are usually singular too, things like "anyone, whatever, whoever, no one, each, every" etc. The only exceptions to this are SANAM: Some, Any, None, All, Most.

If you see the words "each or every" in front of a subject, even if that subject is plural - i.e. Each of the shirts - the verb should be singular - i.e. "each of the shirts is pretty". If each or every comes after the subject it has no bearing!

The phrase "the number of" = singular. The phrase "a number of" = plural. Always.

When in doubt, bet singular.