Speed Time and Distance.

$S = \frac{D}{T}$

$S \propto D$ (Time = const)

$S \propto \frac{1}{T}$ (D = const)

Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken.}}$

Round trip

(D = const)

$S_{one\ side} = a$ m/hr $\qquad S_{other\ side} = b$ m/hr

$D_{one\ side} = D$ miles $\qquad D_{other\ side} = D$ miles

$T_{one\ side} = \frac{D}{a}$ hr. $\qquad T_{other} = \frac{D}{b}$ hr.

$$\text{Avg. speed} = \frac{2D}{\frac{D}{a} + \frac{D}{b}} = \frac{2D}{\frac{D(a+b)}{ab}} = \frac{2ab}{a+b}$$

∴ **Average speed = Harmonic mean of two speeds.**

Ex: 40 km/hr, $D = x$ km; 60 km/hr, $D = x$ km

↳ Avg. speed = $\frac{2 \times 40 \times 60}{60+40} = 48$

* Avg. speed describes the entire journey not the moment within it.

2) Unit conversion:-

* Always maintain similar quantities across the same units.

* Convert quantities via their units.

$36\ \frac{km}{hr} = 36 \times \frac{1000}{60 \times 60} = 10$ m/s.

One body moves vs Two body move.

Question → Read, process, visualize and organize

then take a call

one body | Two body.

One body moves.

* Algebra process ($S = D/T$)
- Read, process, visualize and organize the data in a D S T table.
- Jot down addⁿ info on the side.
- Always make a note of the constant.
- finish the table, define variables for unknown info (Limit them to 1 variable)
- Set up eqⁿs either using distance or using time.
- Use avg. speed wherever applicable (Total dist / Total time) or (Discreet, Harmonic mean)

Qu:- Jane rode her bicycle to the repair shop and rode the bus home by the same route. Including the time, she spent at the shop, she spent a total of 1 hour traveling from her home to the shop and back again. If she rode her bycycle at an average speed of 5 m/hr and the bus travelled at an avg. speed of 20 m/hr; then for how many miles did she ride her bicycle? A) 2 B) 4 C) 5 D) 8 E) 10

Sol:- M1 D = const

	D	S	T
B:	x	5 m/hr	x/5
Bu	x	20 m/hr	x/20

Total time = 1 hr.

$\frac{x}{5} + \frac{x}{20} = 1$

$\therefore \boxed{x = 4.}$

M2

avg. speed = $\frac{2 \times 5 \times 20}{20+5} = \frac{2 \times 5 \times 20^{4}}{25_{5}} = 8.$

Time = 1 hr. $\therefore$ Distance = $8 \times 1 = 8$ miles. (Total)

One side distance = $8/2 = 4$ miles.

Qu:- A car traveling at a certain constant speed takes 2 sec longer to travel 1 km that it would take to travel 1 km at 75 km/hr. At what speed, in km/hr, is the car traveling?

A) 71.5 B) 72 C) 72.5 D) 73 E) 73.5.

Sol:-

	D	S	T
1st	1	?	$\frac{1}{75}$ hr + 2 sec (50 sec)
2nd	1	75 km/hr	$\frac{1}{75}$ hr (48 sec)

Speed = $\frac{1}{50}$ km/sec = $\frac{1}{50} \times 3600$ km/hr = **72 km/hr.**

Ques:- Boat upstream / Downstream. Distance (up / Down) = 90.

$Speed_{up} = (v-3)$ m/hr $\quad Speed_{Do} = (v+3)$ m/hr.

$T_{up} - T_{Do} = \frac{1}{2}$ hr. $\quad Time_{Do} = ?$

Soln:- D = constant

	D	S	T
Up	90	$(v-3)$ m/hr	$\frac{90}{v-3}$
Down	90	$(v+3)$ m/hr	$\frac{90}{v+3}$

$(T_U - T_{Do} = \frac{1}{2})$

$$\frac{90}{v-3} - \frac{90}{v+3} = \frac{1}{2} \Rightarrow \boxed{v = 33}$$

$$T_{Do} = \frac{90}{v+3} = \frac{90}{36} = 2.5$$

* Concept :-

Upstream, $Speed_{up} = S_{boat} - S_{current}$.

Downstream, $Speed_{Doin} = S_{Boat} + S_{current}$.

Ques:- A man rows for 3 hours downstream and then for 3 hours upstream. In this whole process he covers a distance of 12 km. If the speed of the stream is 1 kmph, for how much more time will he have to row upstream to reach the starting point?

A) 6 hrs B) 5 hrs C) 4 hrs D) 3 hrs E) 2 h

Sol:-

	D	S	T
Up		$n-1$	3
Down	9 km	$n+1$	3

~~$T_{up} + T_{Do} = 3$~~.

Total Dist = 12 km.

$$3(n-1) + 3(n+1) = 12.$$

$$6n = 12 \Rightarrow (n = 2)$$

$$\text{Time} = \frac{\text{Distance remain}}{\text{Speed}} = \frac{6}{1} = 6 \text{ hrs.}$$

M-52

Total distance = 12 km.

Total time = 6 hrs.

Avg. speed = 2 km/hr.

S_{up} = 1 km/hr S_{down} = 3 km/hr

$$\text{Time} = \frac{6}{1} = 6 \text{ hr}.$$

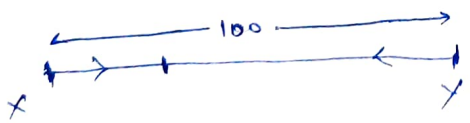
Two bodies movement:

1) Graphically represent the questions
   - set up the graph to represent points from which the two bodies move simultaneously.
   - mark a logical meeting point.
2) Read, process, visualize and organize in a D S T table.
3) Make a note of the constant.
4) Create eqn using either the distance or the time.

Ques:- Two trains started simultaneously from opposite ends of a 100-mile route and traveled towards each other on parallel tracks. Train X, traveling at a constant rate, completed the 100 mile trip in 5 hours; train Y, traveling at a constant rate, completed the 100 mile trip in 3 hr. How many miles had X traveled when it met train Y?

A) 37.5 B) 40 C) 60 D) 62.5 E) 77.5

Sol:-



$S_X = \frac{100}{5} = 20$ m/hr

$S_Y = \frac{100}{3}$ m/hr.

Here, t = [illegible]

	D	S	T
X	20t	20	t
Y	$\frac{100-t}{3}$	$\frac{100}{3}$	t

$$20t + \frac{100-t}{3} = 100$$

$$\Rightarrow \frac{160t}{3} = 100$$

$$20t = \frac{100 \times 3}{8_2} = \frac{75}{2} = 37.5$$

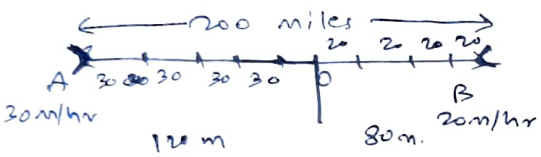
$\therefore$ Distance traveled by X is 37.5 miles

Relative speed

* Cumulative speed of two bodies moving simultaneously w.r.t each other in a particular direction.
* towards → add the speeds.
  Same dir → Substract speed.
* Relative speed with the gap distance (GD)
  GD → distance b/w the two bodies before they start moving simultaneously.

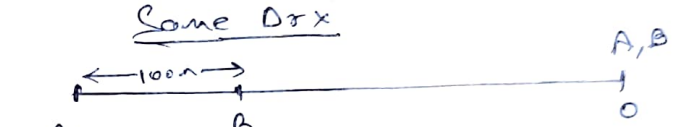
Rel. speed = $\frac{GD}{\text{constant time}}$; constant time it takes for the two bodies to meet.

* Towards



RS = 50 km/hr.
GD = 200 m
T = $\frac{200}{50}$ = 4 hrs.

Same Dir



	A	B
Start	0 ←100→	100
1hr	30 ←90→	120
2hr	60 ←80→	140

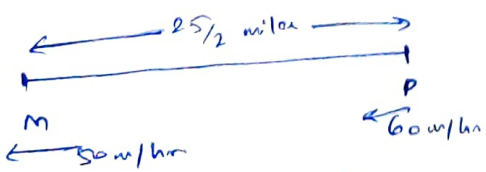
∴ RS = 10 m/hr.
GD = 100
T = 10 hrs

Ques: Mary passed certain gas station on a highway while traveling west at a constant speed of 50 m/hr. Then, 15 min later, Paul passed the same gas station while traveling west at a constant speed of 60 miles/hr. If both drivers maintained their speeds and both remained on the highway for at least 2 hours, how long after he passed the gas station did Paul catch up with Mary?

A) 30 min B) 45 min C) 1 hr D) 1 hr 15 min E) 1 hr 30 min

Soln:

M1



T_m = 15 min.

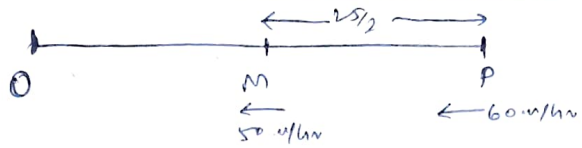
Distance $_m$ = $\frac{15}{60} \times 50 = \frac{25}{2}$ miles

R.S = 10 m/hr.

∴ Time = $\left(\frac{25}{2} \times \frac{1}{10}\right) \times 60^{3}$ (minutes)

= 75 minutes = 1 hr 15 min.

M2.



$D_M = \frac{50 \times 15}{2 \times 60} = \frac{25}{2}$ m.

t = constant

	D	S	T
M	50t	50	t
P	60t	60	t

$\therefore \quad 60t - 50t = \frac{25}{2}$

$\Rightarrow \quad 10t = \frac{25}{2}$

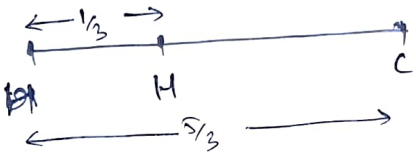
$\Rightarrow \quad t = \frac{5}{4}$ hr.

$\therefore \quad t = 1$ hr 15 min.

Question: A hiker walking at a constant rate of 4 miles/hr is passed by a cyclist traveling in the same direction along the same path at a constant rate of 20 m/hr. The cyclist stops to wait for the hiker 5 minutes after passing her, while the hiker continues to walk at her constant rate. How many minutes must the cyclist wait until the hiker catches up?

Soln:

M1



$D_H = \frac{4}{3}$ miles

$S_H = 4$ m/hr.

$D_H = S_H \times \frac{5}{60} = \frac{4 \times 5}{60} = \frac{1}{3}$ miles.

$\therefore \quad T_H = \frac{4}{3} \times \frac{1}{4}$

$= \frac{1}{3}$ hr.

$D_C = S_C \times \frac{5}{60} = \frac{20 \times 5}{60} = \frac{5}{3}$ miles.

$T_H = 20$ min.

M2.

	D	S	T
H	$4(\frac{5}{60}+t)$	4 m/hr	$\frac{5}{60}+t$
C	$\frac{5}{3}$	20 m/hr	$\frac{5}{60}$

Distance traveled by hiker & cyclist are same.

$\therefore \quad 4\left(\frac{5}{60}+t\right) = \frac{5}{3}$

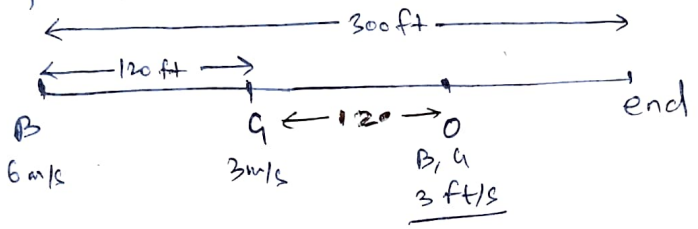
$\Rightarrow \quad \frac{1}{3} + 4t = \frac{5}{3} \quad \Rightarrow \quad 4t = \frac{4}{3} \quad \Rightarrow \quad t = \frac{1}{3}$ hr.

$t = \frac{1}{3} \times 60$ min $= 20$ min

Question The 'moving walkway' is a 300 ft long conveyor belt that moves continuously at 3 feet/sec. When Bill steps on the walkway, a group of people that are also on the walkway stands 120 feet in front of him. He walks towards the group at a combined rate (including both walking & foot speed) of 6 ft/sec, reaches the group of people, and then remains stationary until the walkway ends. What is Bill's average rate of movement for his trip along the moving walkway? A) 2 ft/sec B) 2.5 ft/sec C) 3 ft/sec D) 4 ft/sec E) 5 ft/sec

Sol:→

M1 (Algebra).



	D	S	T
B	6t	6	t
G	3t	3	t

$6t - 3t = 120$

$\therefore t = 40$ sec

$G_{\text{distance in 40 sec}} = 3 \times 40 = 120.$

Distance left $= 300 - 240 = 60.$

Speed $= 3$ ft/s, Time $= \frac{60}{3} = 20$ sec.

Avg. Speed $= \frac{300}{60} \left(\frac{\text{Total Distance}}{\text{Total time}}\right) = 5$ ft/sec.

M2. Relative velocity

Relative velocity $= (6 - 3) = 3$ ft/s

$GD = 120.$

Time $= \frac{120}{3} = 40$ sec.

~~Distance traveled by G in 40 sec = 120 ft.~~

~~Distance left = 60 ft.~~, ~~Speed = 3 ft/sec.~~

~~Time = $\frac{60}{3}$ = 20 sec.~~, ~~Total time = 60 sec.~~

Qu 11 How many hours did it take helen to drive from her house to her parents house?

I. Helen's average speed on this trip was 72 km/hr.

II. If Helen's avg. speed on this trip had been 8 km/hr greater, it would have taken her 1 hour less.

Soln:-

	D	S	T
H	xy km	y km/hr	x hrs

x 1) $y = 72$ km/hr.

2)

	D	S	T
	$(y+8)(x-1)$	$y+8$	$x-1$

Distance are same. $\Rightarrow$ $xy = (y+8)(x-1)$

$xy = xy - y + 8x - 8$

$8x = y + 8$ — Not sufficient

1 & 2 $8x = y+8$ & $y = 72$.

$\therefore$ $x =$ ✓ (C)

Qu Khalil drove 120 km in a certain amount of time. what was his average speed, in km/hr during this time?

I If Khalil had driven at an average speed that was 5 km/hr faster, his driving time would have been reduced by 20 min

II If Khalil had driven at an avg. speed that was 25% faster, his driving time would have been reduced by 20%.

Soln:-

	D	S	T
K	120	$\left(\frac{120}{x}\right)$	x

I

	D	S	T
K	120	$\left(\frac{120}{x}+5\right)$	$\left(x - \frac{1}{3}\right)$

$\Rightarrow$ $120 = \left(\frac{120}{x}+5\right) \times \left(x - \frac{1}{3}\right)$

$\Rightarrow$ $120 = 120 - \frac{120}{3x} + 5x - \frac{5}{3}$.

We can find value of x from here.

$\therefore$ Sufficient.

II

D	S	T
120	$\frac{5}{4}\left(\frac{120}{x}\right)$	$\frac{4}{5}(x)$

$\Rightarrow$ $120 = \frac{5}{4}\left(\frac{120}{x}\right) \times \frac{4}{5}(x)$ $\Rightarrow$ $120 = 120$.

Not Sufficient.