Triangle

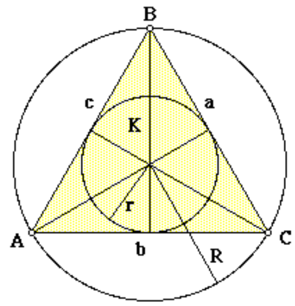
A polygon (plane figure) with 3 angles and 3 sides.

Sides: a, b, c
Opposite angles: A, B, C

Altitudes: h_a , h_b , h_c
Medians: m_a , m_b , m_c
Angle bisectors: t_a , t_b , t_c

Perimeter: P
Semiperimeter: s
Area: K

Radius of circumscribed circle: R
Radius of inscribed circle: r



Equilateral Triangle

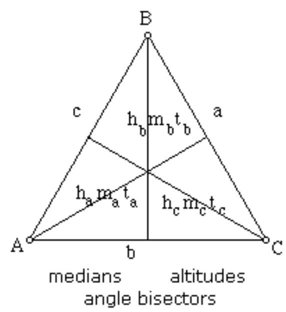
A triangle with all three sides of equal length.

a = b = c.
A = B = C = Pi/3 radians = 60^o

P = 3a
s = 3a/2
K = a^2sqrt(3)/4

$h_a = m_a = t_a$ = a sqrt(3)/2

R = a sqrt(3)/3
r = a sqrt(3)/6

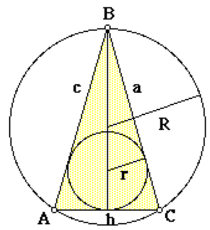


Isosceles Triangle

A triangle with two sides of equal length.

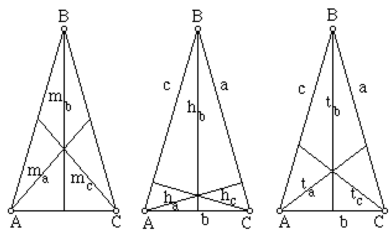
$a = c$
$A = C$

$B + 2A = \text{Pi radians} = 180^\circ$
$P = 2a + b$
$s = a + b/2$
$K = b\ \text{sqrt}(4a^2 - b^2)/4 = a^2 \sin(B)/2 = ab \sin(A)/2$



$h_a = b\ \text{sqrt}(4a^2 - b^2)/(2a) = a \sin(B) = b \sin(A)$
$m_a = \text{sqrt}(a^2 + 2b^2)/2$
$t_a = b\ \text{sqrt}(a[2a+b])/(a+b) = b \sin(A)/\sin(3A/2)$
$h_b = m_b = t_b = \text{sqrt}(4a^2 - b^2)/2 = a \cos(B/2)$

$R = a^2 b/4K = a/[2 \sin(A)] = b/[2 \sin(B)]$
$r = K/s = b\ \text{sqrt}[(2a-b)/(2a+b)]/2$



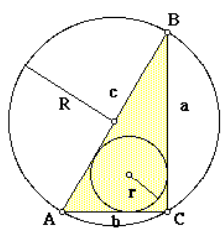
medians altitudes angle bisectors

Right Triangle

A triangle with one right angle.

$C = A + B = \text{Pi}/2 \text{ radians} = 90^\circ$

$c^2 = a^2 + b^2$
(Pythagorean Theorem)



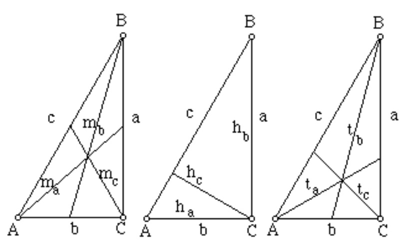
$P = a + b + c$
$s = (a+b+c)/2$
$K = ab/2$

$h_a = b$
$h_b = a$
$h_c = ab/c$
$m_a = \text{sqrt}(4b^2 + a^2)/2$
$m_b = \text{sqrt}(4a^2 + b^2)/2$
$m_c = c/2$

$t_a = 2bc \cos(A/2)/(b+c) =$
$\text{sqrt}[bc(1 - a^2/[b+c]^2)]$

$t_b = 2ac \cos(B/2)/(a+c) =$
$\text{sqrt}[ac(1 - b^2/[a+c]^2)]$

$t_c = ab\ \text{sqrt}(2)/(a+b)$



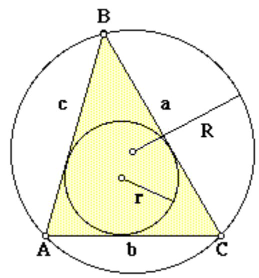
medians altitudes angle bisectors

$R = c/2$
$r = ab/(a+b+c) = s - c$

Scalene Triangle

A triangle with no two sides equal.

(Note that the following formulas work with all triangles, not just scalene triangles.)



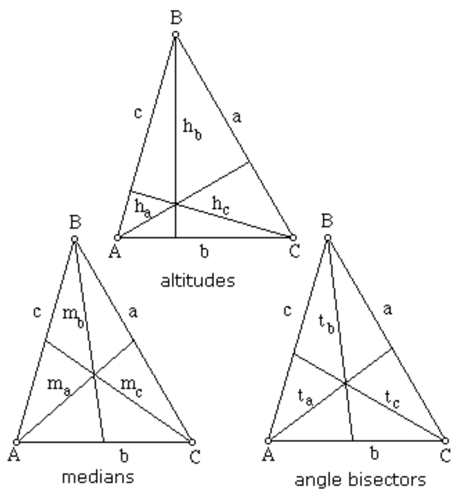
$P = a + b + c$
$s = (a+b+c)/2$

$K = ah_a/2 = ab \sin(C)/2 =$
$a^2 \sin(B) \sin(C)/[2 \sin(A)] =$
$\text{sqrt}[s(s-a)(s-b)(s-c)]$
(Heron's or Hero's Formula)

$h_a = c \sin(B) = b \sin(C) = 2K/a$
$m_a = \text{sqrt}(2b^2+2c^2-a^2)/2$
$t_a = 2bc \cos(A/2)/(b+c) =$
$\text{sqrt}[bc(1-a^2/[b+c]^2)]$

$R = abc/4K =$
$a/[2 \sin(A)] =$
$b/[2 \sin(B)] =$
$c/[2 \sin(C)]$

$r = 2K/P = K/s =$
$\text{sqrt}[(s-a)(s-b)(s-c)/s] =$
$c \sin(A/2)\sin(B/2)/\cos(C/2) =$
$ab \sin(C)/(2 s) =$
$(s-c)\tan(C/2)$



Quadrilateral

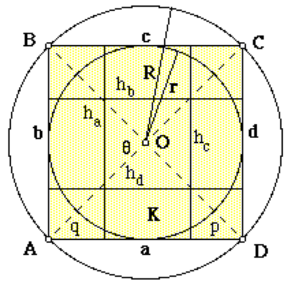
A polygon (plane figure) with 4 angles and 4 sides.

Sides: a, b, c, d
Angles: A, B, C, D
Around the quadrilateral are a, A, b, B, c, C, d, D, and back to a, in that order

Altitudes: h_a , etc.
Diagonals: p = BD, q = AC, intersect at O
Angle between diagonals: theta

Perimeter: P
Semiperimeter: s
Area: K

Radius of circumscribed circle: R
Radius of inscribed circle: r



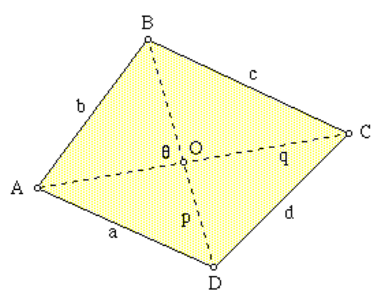
General

P = a + b + c + d.
s = P/2 = (a+b+c+d)/2

A + B + C + D = 2 Pi radians = 360°

K = pq sin(theta)/2
K = $(b^2+d^2-a^2-c^2)$tan(theta)/4
K = sqrt[$4p^2q^2-(b^2+d^2-a^2-c^2)^2$]/4

K = sqrt[(s-a)(s-b)(s-c)(s-d)-abcd $\cos^2$([A+C]/2)]
(Bretschneider's Formula)

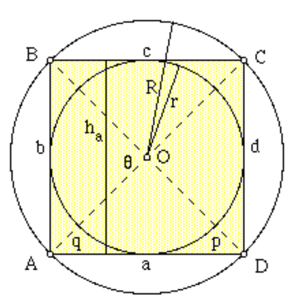


Square

A quadrilateral with four right angles and all four sides of equal length.

a = b = c = d
A = B = C = D = Pi/2 radians = 90°
theta = Pi/2 radians = 90°

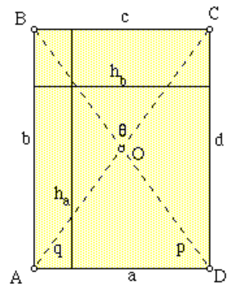
h_a = a
p = q = a sqrt(2)
P = 4a
s = 2a
K = a^2
R = a sqrt(2)/2
r = a/2



Rectangle

A quadrilateral with adjacent sides perpendicular (all four angles are therefore right angles).

a = c, b = d.
A = B = C = D = Pi/2 radians = 90°

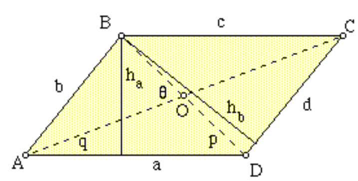


$h_a = b$
$h_b = a$
$p = q = \text{sqrt}(a^2+b^2)$
theta = 2 arctan(a/b)
P = 2(a+b)
s = a + b
K = ab
$R = p/2 = \text{sqrt}(a^2+b^2)/2$
r = minimum(a,b)/2

Parallelogram

A quadrilateral with opposite sides parallel.

a = c, b = d
A = C, B = D



A + B = Pi radians = 180°
$h_a = b \sin(A) = b \cos(B\text{-Pi}/2)$
$h_b = a \sin(A) = a \cos(B\text{-Pi}/2)$

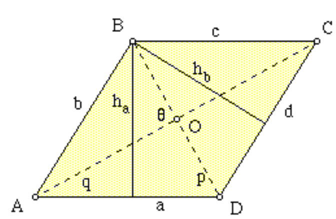
$p = \text{sqrt}[a^2+b^2\text{-}2ab \cos(A)]$
$q = \text{sqrt}[a^2+b^2\text{-}2ab \cos(B)]$
$p^2+q^2 = 2(a^2+b^2)$
$\text{theta} = \arccos([a^2\text{-}b^2]/pq)$
P = 2*(a+b)

s = a + b
K = ab sin(A) = ab sin(B) =
bh_b
= pq sin(theta)/2

Rhombus

A parallelogram with all sides equal.

a = b = c = d
A = C, B = D
theta = Pi/2 radians = 90°

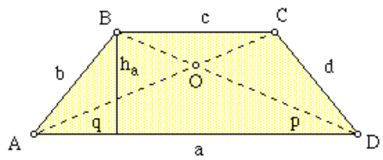


A + B = Pi radians = 180°
$h_a = a \sin(A) = a \cos(B\text{-Pi}/2)$
$h_a = h_b$

$p = a\ \text{sqrt}[2\text{-}2 \cos(A)]$
$q = a\ \text{sqrt}[2\text{-}2 \cos(B)]$

$p^2+q^2 = 4a^2$
P = 4a
s = 2a
$K = a^2\sin(A) = a^2\sin(B)$
$= ah_a = pq/2$

Trapezoid (American) Trapezium (British)*



a parallel to c, $m = (a+c)/2$
$A + B = C + D = $ Pi radians $= 180°$

$$P = a + b + c + d$$

$$K = h_a m = h_a(a+c)/2$$

If $a = c$, the trapezoid is actually a parallelogram, so $b = d$, and the height and area cannot be determined from a, b, c, and d alone. If a and c are not equal, then

$$h_a^2 = (a+b-c+d)(-a+b+c+d)(a-b-c+d)(a+b-c-d)/[4(a-c)^2].$$

If $h_a^2 < 0$, no trapezoid having those side lengths exists.

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*From *The Words of Mathematics* by Steven Schwartzman (1994, Mathematical Association of America):

trapezoid (noun); **trapezoidal** (adjective); **trapezium**, plural *trapezia* (noun): The Greek word *trapeza* "table" was composed of *tetra* "four" and the Indo-European root *ped-* "foot." A Greek table must have had four feet (= legs). The suffix *-oid* (*q.v.*) means "looking like," so that a trapezoid is a figure that looks like a table (at least in somebody's imagination). Some Americans define a trapezoid as a quadrilateral with at least one pair of parallel sides. Under that definition, a parallelogram is a special kind of trapezoid. For other Americans, however, a trapezoid is a quadrilateral with one and only one pair of parallel sides, in which case a parallelogram is not a trapezoid. The situation is further confused by the fact that in Europe a trapezoid is defined as a quadrilateral with no sides equal. Even more confusing is the existence of the similar word *trapezium,* which in American usage means "a quadrilateral with no sides equal," but which in European usage is a synonym of what Americans call a trapezoid. Apparently to cut down on the confusion, trapezium is not used in American textbooks. The trapeze used in a circus is also related, since a trapeze has or must once have had four "sides": two ropes, the bar at the bottom, and a support bar at the top.

Kite

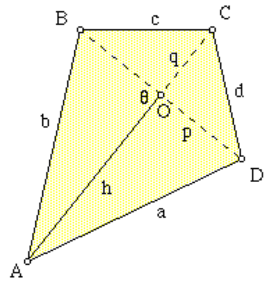
A quadrilateral with two pairs of distinct adjacent sides equal in length.

$a = b$, $c = d$
theta = Pi/2 radians = 90^o

OB = OD = p/2, OA = h, OC = q - h

$h = \text{sqrt}(a^2-p^2/4)$
$q = \text{sqrt}(a^2-p^2/4) + \text{sqrt}(c^2-p^2/4)$

$P = 2(a+c)$
$K = pq/2$



Cyclic Quadrilateral

A quadrilateral all of whose vertices lie on a circle.

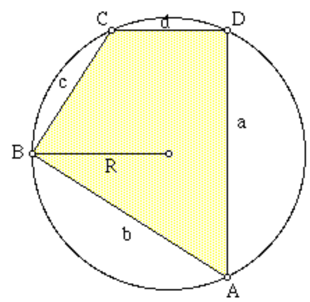
Points A, B, C, and D lie on a circle of radius R.
A + C = B + D = Pi radians = 180^o

K = sqrt[(s-a)(s-b)(s-c)(s-d)]
(Brahmagupta's Formula)
K = sqrt[(ab+cd)(ac+bd)(ad+bc)]/4*R

p = sqrt[(ac+bd)(ad+bc)/(ab+cd)]
q = sqrt[(ab+cd)(ac+bd)/(ad+bc)]

R = sqrt[(ab+cd)(ac+bd)(ad+bc) / (s-a)(s-b)(s-c)(s-d)]/4

theta = arcsin[2K/(ac+bd)]



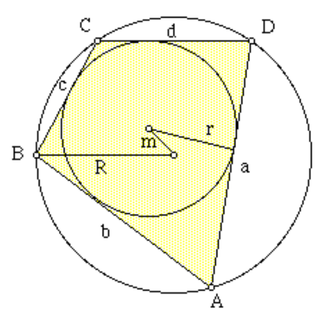
Cyclic-Inscriptable

A quadrilateral within which a circle can be inscribed, tangent to all four sides.

Points A, B, C, and D lie on a circle of radius R.
Sides a, b, c, and d are tangent to a circle of radius r.
m = distance between the centers of the two circles.

A + C = B + D = Pi radians = 180^o
a + c = b + d
K = sqrt[abcd]

r = sqrt[abcd]/s
R = sqrt[(ab+cd)(ac+bd)(ad+bc)/abcd]/4
$1/(R+m)^2 + 1/(R-m)^2 = 1/r^2$

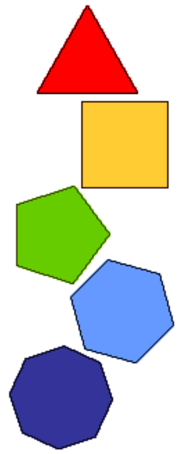


Regular Polygon

Number of sides, all equal length a: n
Number of interior angles, all equal measure beta: n
Central angle subtending one side: alpha

Perimeter: P
Area: K

Radius of circumscribed circle: R
Radius of inscribed circle: r



beta = Pi(n-2)/n radians = 180^o(n-2)/n
alpha = 2 Pi/n radians = 360^o/n
alpha + beta = Pi radians = 180^o

P = na = 2nR sin(alpha/2)
K = na^2 cot(alpha/2)/4
= nR^2 sin(alpha)/2
= nr^2 tan(alpha/2)
= na sqrt($4R^2$-a^2)/4

R = a csc(alpha/2)/2
r = a cot(alpha/2)/2

a = 2r tan(alpha/2) = 2R sin(alpha/2)

Special Cases of the Regular Polygon

n = 3 (equilateral triangle)
n = 4 (square)
n = 5 (regular pentagon)
n = 6 (regular hexagon)
n = 8 (regular octagon)

The Regular Pentagon

Number of sides n = 5
Internal angles beta = $3\pi/5$ radians = 108 degrees
Central angles alpha = $2\pi/5$ radians = 72 degrees

Perimeter P = 5a = 5R sqrt(10-2 sqrt[5])/2
Area K = $5a^2$ sqrt(1+2/sqrt[5])/4
= $5R^2$ sqrt(10+2 sqrt[5])/8
= $5r^2$ sqrt(5-2 sqrt[5])
= 5a sqrt($4R^2$-a^2)/4

Circumradius R = a sqrt(2+2/sqrt[5])/2
Apothem r = a sqrt(1+2/sqrt[5])/2 = R(1+sqrt[5])/4

Side a = 2r sqrt(5-2 sqrt[5]) = R sqrt(10-2 sqrt[5])/2

The Regular Hexagon 

Number of sides $n = 6$
Internal angles beta $= 2\pi/3$ radians $= 120$ degrees
Central angles alpha $= \pi/3$ radians $= 60$ degrees

Perimeter $P = 6a = 6R$
Area $K = 3a^2 \text{ sqrt}(3)/2$
$= 3R^2 \text{ sqrt}(3)/2$
$= 2r^2 \text{ sqrt}(3)$
$= 3a \text{ sqrt}(4R^2\text{-}a^2)/2$

Circumradius $R = a$
Apothem $r = a \text{ sqrt}(3)/2 = R \text{ sqrt}(3)/2$

Side $a = 2r/\text{sqrt}(3) = R$

The Regular Octagon 

Number of sides $n = 8$
Internal angles beta $= 3\pi/4$ radians $= 135$ degrees
Central angles alpha $= \pi/4$ radains $= 45$ degrees

Perimeter $P = 8a = 8R \text{ sqrt}(2\text{-sqrt}[2])$
Area $K = 2a^2(\text{sqrt}[2]+1)$
$= 2R^2 \text{ sqrt}(2)$
$= 8r^2(\text{sqrt}[2]\text{-}1)$
$= 2a \text{ sqrt}(4R^2\text{-}a^2)$

Circumradius $R = a \text{ sqrt}(\text{sqrt}[2]/2+1)$
Apothem $r = a(\text{sqrt}[2]+1)/2 = R \text{ sqrt}(2+\text{sqrt}[2])/2$

Side $a = 2r(\text{sqrt}[2]\text{-}1) = R \text{ sqrt}(2\text{-sqrt}[2])$

Constructible Regular Polygons

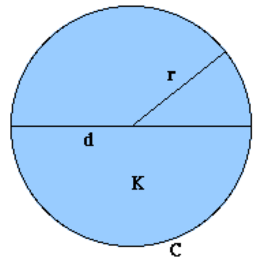
Here is a table of regular polygons constructible with straightedge and compass whose angles are whole numbers of degrees:

n	alpha	beta	a/R
3	120	60	1.7320508 = sqrt(3)
4	90	90	1.4142136 = sqrt(2)
5	72	108	1.1755705 = sqrt(5-sqrt[5])/2
6	60	120	1.0000000 = 1
8	45	135	0.7653669 = sqrt(2-sqrt[2])
10	36	144	0.6180340 = (sqrt[5]-1)/2
12	30	150	0.5176381 = (sqrt[3]-1)/sqrt(2)
15	24	156	0.4158234 = (sqrt[3]sqrt[1-sqrt(5)]+sqrt[2]sqrt[5+sqrt(5)])/4
20	18	162	0.3128689 = (sqrt[2][1+sqrt(5)] - 2 sqrt[5-sqrt(5)])/4
24	15	165	0.2610524 = sqrt([4-sqrt(2)-sqrt(6)]/2)
30	12	168	0.2090569 = (sqrt[6]sqrt[5-sqrt(5)]-sqrt[5]-1)/4
40	9	171	0.1569182 = (sqrt[2]sqrt[2+sqrt(2)][1-sqrt(5)]+ 2 sqrt[2-sqrt(2)]sqrt[5+sqrt(5)])/(4 sqrt[2])
60	6	174	0.1046719 = ([sqrt(3)+1][sqrt(5)-1]sqrt[2]+ 2[1-sqrt(3)]sqrt[5+sqrt(5)])/8
120	3	177	0.0523539 = ([sqrt(2-sqrt[2])+ sqrt(6+3 sqrt[2])][1+sqrt(5)-sqrt(30-6 sqrt[5])]+ [sqrt(2+sqrt[2])-sqrt(6-3 sqrt[2])][sqrt(3)+ sqrt(15)+sqrt(10-2 sqrt[5])])/16

Circle

All points on the circumference of a circle are equidistant from its center.

Radius: r
Diameter: d
Circumference: C
Area: K

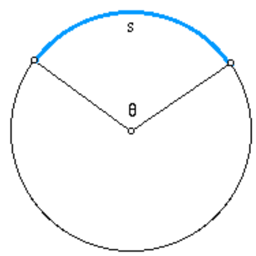


$d = 2r$
$C = 2 \pi r = \pi d$

$K = \pi r^2 = \pi d^2/4$
$C = 2\, \mathrm{sqrt}(\pi K)$

$K = C^2/4\pi = Cr/2$

Arc of a Circle



A curved portion of a circle.

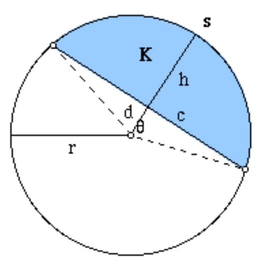
Length: s

Central angle:
theta (in radians),
alpha (in degrees)

$s = r\,\theta = r\,\alpha \pi/180$

Segment of a Circle

Either of the two regions into which a secant or a chord cuts a circle. (However, the formulas below assume that the segment is no larger than a semi-circle.)



Chord length: c
Height: h

Distance from center of circle to chord's midpoint: d
Central angle: theta (in radians), alpha (in degrees)

Area: K
Arc length: s

theta = 2 arccos(d/r) = 2 arctan(c/(2d)) = 2 arcsin(c/(2r))
h = r - d

c = 2 sqrt(r^2-d^2) = 2r sin(theta/2) = 2d tan(theta/2) = 2 sqrt[h(2r-h)]
d = sqrt($4r^2$-c^2)/2 = r cos(theta/2) = c cot(theta/2)/2

K = r^2[theta-sin(theta)]/2 = r^2arccos([r-h]/r) - (r-h)sqrt(2rh-h^2)
= r^2arccos(d/r) - d sqrt(r^2-d^2)

theta = s/r
K = r^2[s/r - sin(s/r)]/2

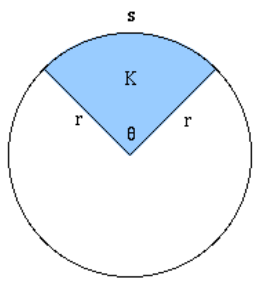
Sector of a Circle

The pie-shaped piece of a circle 'cut out' by two radii.

Central angle:
theta (in radians),
alpha (in degrees)

Area: K
Arc length: s

$K = r^2$theta/2 $= r^2$alpha Pi/360
theta = s/r
K = rs/2

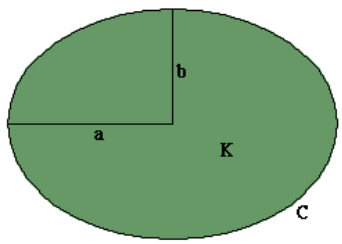


"A conic (or conic section) is a plane curve that can be obtained by intersecting a cone with a plane that does not go through the vertex of the cone. There are three possibilities, depending on the relative position of the cone and the plane. If no line of the cone is parallel to the plane, the intersection is a closed curve, called an **ellipse**. If one line of the cone is parallel to the plane, the intersection is an open curve whose two ends are asymptotically parallel; this is called a **parabola**. Finally, there may be two lines in the cone parallel to the plane; the curve in this case has two open pieces, and is called a **hyperbola**."

Ellipse

Semi-axes: a, b

Eccentricity: $e = \text{sqrt}(a^2-b^2)/a$
Area: K
Circumference: C



K = Pi ab

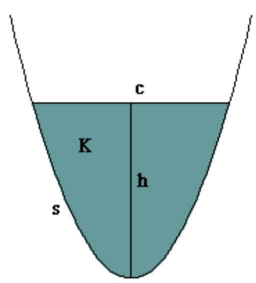
C = 4aE, where E is an elliptic integral with k = e, which can be used to derive the following formulas:

$C = \pi(a+b)[1 + x^2/4 + x^4/64 + \ldots]$, where $x = (a-b)/(a+b)$

$C = \pi(a+b)(1 + 3x^2/[10 + \text{sqrt}(4 - 3x^2)])$, approximately

Segment of a Parabola

Height: h
Chord length: c
Area: K
Length: s



$s = c[1+2(2h/c)^2/3-2(2h/c)^4/5+\ldots]$

$s = \text{sqrt}[4h^2+c^2/4]+[c^2/(8h)] \ln[(2h+\text{sqrt}[4h^2+c^2/4])/(c/2)]$

K = 2ch/3
K = 4T/3, where T is the area of the triangle formed by the chord and the point of tangency of a tangent to the parabola parallel to the chord

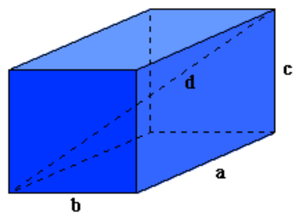
A parallelepiped (alternate spelling parallelopiped) is a polyhedron with six faces bounded by three pairs of parallel planes, so all its faces are parallelograms. It is also a prism the base of which is a parallelogram.

Rectangular Parallelepiped

A three-dimensional figure all of whose face angles are right angles, so all its faces are rectangles and all its dihedral angles are right angles. (A dihedral angle is an angle created by two intersecting planes.)

Edges: a, b, c
Diagonal: d
Total surface area (total area of all the faces of the figure): T
Volume: V

$d = \text{sqrt}(a^2+b^2+c^2)$
$T = 2(ab+ac+bc)$
$V = abc$
Face diagonals $\text{sqrt}(a^2+b^2)$, $\text{sqrt}(a^2+c^2)$, $\text{sqrt}(b^2+c^2)$



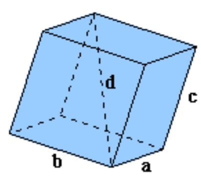
Cube

A three-dimensional figure with six congruent square sides.

$a = b = c$

$d = a\ \text{sqrt}(3)$
$T = 6a^2$
$V = a^3$

Face diagonal $a\ \text{sqrt}(2)$

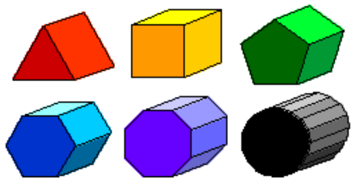


Prism

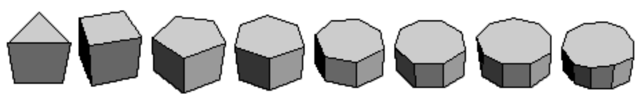
A polyhedron with two congruent, parallel bases that are polygons, and all remaining faces parallelograms.

Height: h
Area of base: B
Length of lateral edge: l
Area of right section: A
Perimeter of right section: P

Lateral surface area: S
Volume: V



$S = lP$
$V = hB = lA$



Pyramid

A pyramid is a polyhedron of which one side, the base, is a polygon (not necessarily a regular polygon), and all the rest are triangles sharing a common point, the vertex. A pyramid is regular if the base is a regular polygon and the other faces are congruent isosceles triangles

Height: h
Area of base: B
Slant height: s (regular pyramid)
Perimeter of base: P
Lateral surface area: S
Volume: V

$S = sP/2$ (regular pyramid)

$V = hB/3$

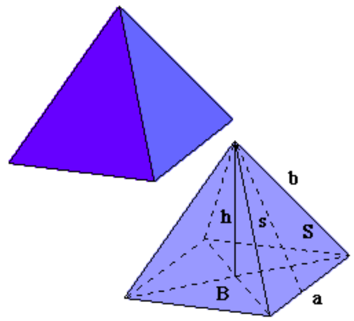
Square Pyramid

The base is a square, and all triangular faces are congruent isosceles triangles.

Side of base: a
Other edges: b
Height: h
Slant height: s

Vertex angle of faces: alpha
Base angle of faces: theta
Base-to-face dihedral angle: beta
Face-to-face dihedral angle: phi

Lateral surface area: S
Total surface area (including base): T
Volume: V



a = sqrt[$2(b^2-h^2)$] = 2 sqrt(b^2-s^2) = 2 sqrt(s^2-h^2)
b = sqrt($h^2+a^2/2$) = sqrt($s^2+a^2/4$) = sqrt($2s^2-h^2$)
h = sqrt($b^2-a^2/2$) = sqrt($s^2-a^2/4$) = sqrt($2s^2-b^2$)
s = sqrt($b^2-a^2/4$) = sqrt($h^2+a^2/4$) = sqrt[$(b^2+h^2)/2$]
theta = arccos(a/2b) = arcsin(s/b) = arctan(2s/a)
alpha = arccos(h^2/b^2) = arcsin(as/b^2) = arctan(as/h^2)
beta = arccos(a/2s) = arcsin(h/s) = arctan(2h/a)
phi = arccos($-a^2/4s^2$) = arcsin(bh/s^2) = arctan($-4bh/a^2$)

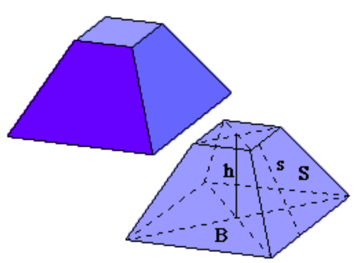
S = 2as
T = a(2s+a)
V = $a^2h/3$

Frustum of a Pyramid

The portion of a pyramid that lies between the base and a plane cutting through it parallel to the base.

Height: h
Area of bases: B_1, B_2
Slant height: s (regular pyramid)
Perimeter of bases: P_1, P_2

Lateral surface area: S
Volume: V



S = s(P_1+P_2)/2
(regular pyramid)

V = h(B_1+B_2+sqrt[B_1B_2])/3

Regular Polyhedron

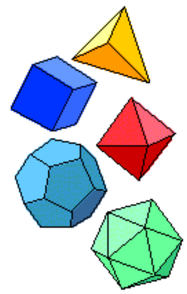
A solid, three-dimensional figure each face of which is a regular polygon with equal sides and equal angles. Every face has the same number of vertices, and the same number of faces meet at every vertex. An inscribed (inside) sphere touches the center of every face, and a circumscribed sphere (outside) touches every vertex.

There are five and only five of these figures, also called the Platonic Solids: the tetrahedron, cube, octahedron, dodecahedron, and icosahedron.

Number of vertices: v
Number of edges: e
Number of faces: f

Edge: a
Radius of circumscribed sphere: R
Radius of inscribed sphere: r

Surface area: S
Volume: V
Dihedral angle between faces: delta (in degrees)

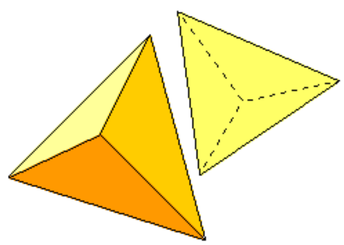


Tetrahedron

A three-dimensional figure with 4 equilateral triangle faces, 4 vertices, and 6 edges.

v = 4, e = 6, f = 4
a = (2 sqrt[6]/3)R
r = (1/3)R
R = (sqrt[6]/4)a
S = sqrt(3)a^2
V = (sqrt[2]/12)a^3
delta = arccos(1/3) = 70° 32'

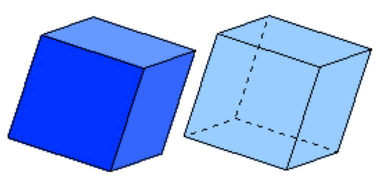
h = height or altitude
h = (sqrt[6]/3)a



Cube

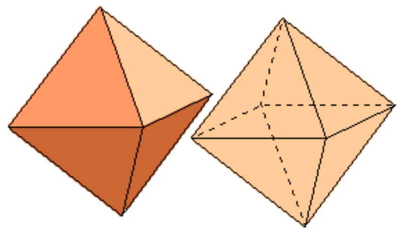
A three-dimensional figure with 6 square faces, 8 vertices, and 12 edges.

v = 8, e = 12, f = 6
a = (2 sqrt[3]/3)R
r = (sqrt[3]/3)R
R = (sqrt[3]/2)a
r = (1/2)a
S = 6 a^2
V = a^3
delta = arccos(0) = 90°



Octahedron

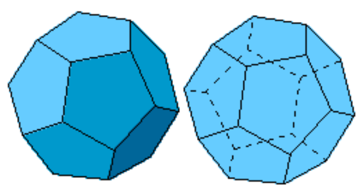
A three-dimensional figure with 8 equilateral triangle faces, 6 vertices, and 12 edges.



v = 6, e = 12, f = 8
a = sqrt(2)R
r = (sqrt[3]/3)R
R = (sqrt[2]/2)a
r = (sqrt[6]/6)a
S = 2 sqrt(3)a^2
V = (sqrt[2]/3)a^3
delta = arccos(-1/3) = 109° 28'

Dodecahedron

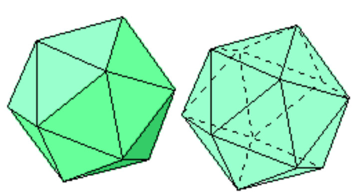
A three-dimensional figure with 12 regular pentagon faces, 20 vertices, and 30 edges.



v = 20, e = 30, f = 12
a = ([sqrt(5)-1]sqrt[3]/3)R
r = (sqrt[75+30 sqrt(5)]/15) R
R = (sqrt[3][1+sqrt[5])/4)a
r = (sqrt[250+110 sqrt (5)]/20)a
S = 3 sqrt(25+10 sqrt[5])a^2
V = ([15+7 sqrt(5)]/4)a^3
delta = arccos(-sqrt[5]/5) = 116° 34'

Icosahedron

A three-dimensional figure with 20 equilateral triangle faces, 12 vertices, and 30 edges.



v = 12, e = 30, f = 20
a = (sqrt[50-10 sqrt(5)]/5)R
r = (sqrt[75+30 sqrt(5)]/15) R
R = (sqrt[10+2 sqrt(5)]/4)a
r = (sqrt[42+18 sqrt(5)]/12)a
S = 5 sqrt(3)a^2
V = (5[3+sqrt(5)]/12)a^3
delta = arccos(-sqrt[5]/3) = 138° 11'

A cylinder is a surface generated by a family of all lines parallel to a given line (the generatrix) and passing through a curve in a plane (the directrix). A right section is the curve formed by the intersection of the surface and a plane perpendicular to the generatrix. The parallel bases of a cylinder may form any angle with the axis.

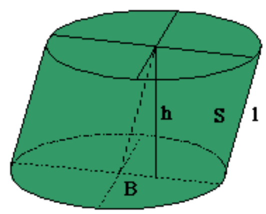
More commonly, a cylinder includes the solid enclosed by a cylinder and two parallel planes. The region of either of the parallel planes enclosed by the surface is called a base of the cylinder. The perpendicular distance between the planes of the bases is the height of the cylinder. The line segment cut on any of the generating lines by the two parallel planes is called a lateral edge.

Circular Cylinder

A cylinder whose bases are circles. The line connecting the centers of the bases is called the axis.

Height: h
Area of base: B
Length of lateral edge: l
Area of *right section*: A
Perimeter of *right section*: P

Lateral surface area: S
Total surface area: T
Volume: V

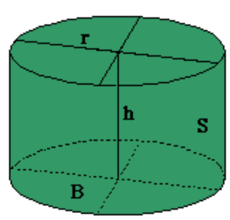


$S = lP$
$T = lP + 2B$
$V = hB = lA$

Right Circular Cylinder

A circular cylinder in which the axis is perpendicular to the bases. (If the axis of a circular cylinder is not perpendicular to the bases, it is called an oblique circular cylinder.)

Height: h
Radius of base: r
Lateral surface area: S
Total surface area: T
Volume: V



$S = 2 \text{ Pi } rh$
$T = 2 \text{ Pi } r(r+h)$
$V = \text{Pi } r^2h$
$A = B = \text{Pi } r^2$
$P = 2 \text{ Pi } r$
$l = h$

Cone

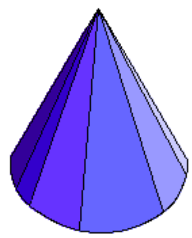
A cone is a surface generated by a family of all lines through a given point (the vertex) and passing through a curve in a plane (the directrix). More commonly, a cone includes the solid enclosed by a cone and the plane of the directrix. The region of the plane enclosed by the directrix is called a base of the cone. The perpendicular distance from the vertex to the plane of the base is the height of the cone.

Height: h
Area of base: B
Volume: V

V = hB/3

Circular Cone

A cone whose base is a circle. The line connecting the center of the base to the vertex is called the axis of the circular cone.

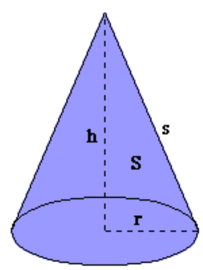


Right Circular Cone

In a right circular cone, the axis is perpendicular to the base. (If the axis of a circular cone is not perpendicular to the base, it is called an oblique circular cone.) The length of any line segment connecting the vertex to the directrix is called the slant height of the cone.

Height: h
Radius of base: r
Slant height: s
Lateral surface area: S
Total surface area: T
Volume: V

$B = \text{Pi}\, r^2$
$s = \text{sqrt}[r^2+h^2]$
$S = \text{Pi}\, rs$
$T = \text{Pi}\, r(r+s)$
$V = \text{Pi}\, r^2h/3$

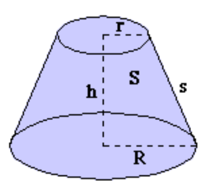


Frustum of a Right Circular Cone

The part of a right circular cone between the base and a plane parallel to the base whose distance from the base is less than the height of the cone.

Height: h
Radius of bases: r, R
Slant height: s
Lateral surface area: S
Total surface area: T
Volume: V

$s = \text{sqrt}([R-r]^2+h^2)$
$S = \text{Pi}(r+R)s$
$T = \text{Pi}(r[r+s]+R[R+s])$
$V = \text{Pi}(R^2+rR+r^2)h/3$



Sphere

A three-dimensional figure with all of its points equidistant from its center.

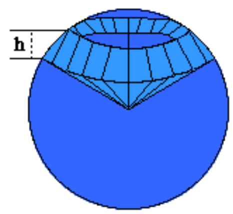
Radius: r
Diameter: d
Surface area: S
Volume: V

$S = 4\ \pi r^2 = \pi d^2$
$V = (4\ \pi/3)r^3 = (\pi/6)d^3$

Sector of a Sphere

The part of a sphere between two right circular cones that have a common vertex at the center of the sphere, and a common axis. (The interior cone may have a base with zero radius.)

Radius: r
Height: h
Volume: V

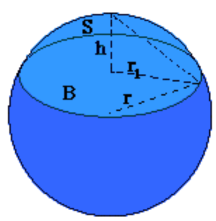


$S = 2\ \pi rh$
$V = (2\ \pi/3)r^2h$

Spherical Cap

The portion of a sphere cut off by a plane. If the height, the radius of the sphere, and the radius of the base are equal: $h = r\ (= r_1)$, the figure is called a hemisphere.

Radius of sphere: r
Radius of base: r_1
Height: h
Surface area: S
Volume: V

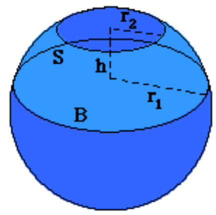


$r = (h^2+r_1^2)/(2h)$
$S = 2\ \pi rh$
$V = (\pi/6)(3r_1^2+h^2)h$

Segment and Zone of a Sphere

Segment: the portion of a sphere cut off by two parallel planes.
Zone: the curved surface of a spherical segment.

Radius of sphere: r
Radii of bases: r_1, r_2
Height: h
Surface area: S
Volume: V

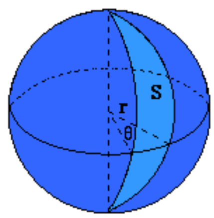


$S = 2 \text{ Pi } rh$
$V = (Pi/6)(3r_1^2+3r_2^2+h^2)h$

Lune of a Sphere

The curved surface of the intersection of two hemispheres.

Radius: r
Central dihedral angle: theta (in radians),
alpha (in degrees)
Surface area: S
Volume enclosed by the lune and the two planes: V

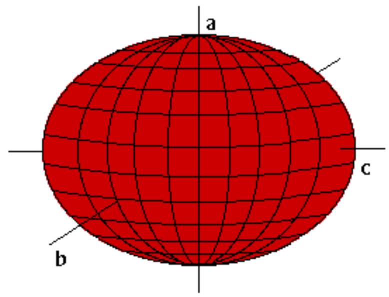


$S = 2r^2\text{theta} = (Pi/90)r^2\text{alpha}$
$V = (2/3)r^3\text{theta} = (Pi/270)r^3\text{alpha}$

Ellipsoid

A three-dimensional figure all planar cross-sections of which are either ellipses or circles.

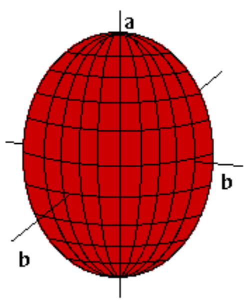
Semi-axes: a, b, c (the semi-axis is half the length of the axis, and corresponds to the radius of a sphere)
Volume: V
V = (4 Pi/3)abc



Prolate Spheroid

Semi-axes: a, b, b (a > b)
Surface area: S

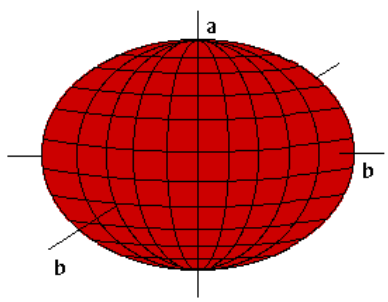
S = 2 Pi b(b+a arcsin[e]/e),
where e = sqrt(a^2-b^2)/a



Oblate Spheroid

Semi-axes: a, b, b (a < b)
Surface area: S

S = 2 Pi b(b+a arcsinh[be/a]/[be/a]),
where e = sqrt(b^2-a^2)/b

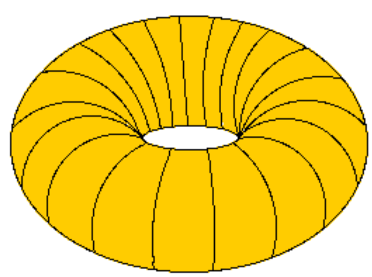


Circular or Ring Torus

The surface of a three-dimensional figure shaped like a doughnut.

Major radius (of the large circle): R
Minor radius (of circular cross-section): r
Surface area: S
Volume: V

S = 4 Pi^2Rr
V = 2 Pi^2Rr^2

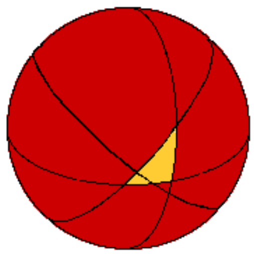


Spherical Polygon

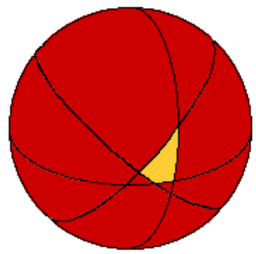
A closed geometric figure on the surface of a sphere formed by the arcs of great circles.

Radius: r
S = (theta-[n-2]Pi)r^2 = (alpha-180[n-2])Pi r^2/180

Number of sides: n
Sum of Angles: theta (in radians),
alpha (in degrees)
Surface area: S



spherical triangle



spherical quadrilateral